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Deformation of singular Lagrangian subvarieties. (English summary)

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The authors develop the first steps of the deformation theory of singular Lagrangian subvarieties. They introduce a complex whose first cohomology computes infinitesimal deformations, while the second cohomology contains information concerning obstructions. They introduce a condition guaranteeing that the complex is constructible: for all k the dimension of the set of points with embedding codimension k is at most k . This is the Lagrangian equivalent of isolated singularity.

Explicit examples are computed. For plane curves it is shown that the Lagrangian Tyurina number equals the Milnor number. Other examples considered are the open swallowtails.

Reviewed by *Jan Stevens*

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Note: This list reflects references listed in the original paper as accurately as possible with no attempt to correct errors.

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